
Quod est primum in compositione, est ultimum in resolutione. Notes on analysis and synthesis in Late Antiquity

*Quod est primum in compositione, est ultimum in resolutione. Notas acerca
de las nociones de 'análisis' y 'síntesis' en la antigüedad tardía*

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Abstract: This article investigates the Greek and Latin origins of the Scholastic dictum that that which is first in composition (equivalent to Greek *sunthesis*) is last in resolution (*Greek analysis*). I study the tradition that synthesis proceeds from prior to posterior, while analysis moves from posterior to prior. First I seek the origins of these notions in Greek mathematics. I then move on to Middle Platonist authors who identify analysis with Platonic *diairesis*, and I next move on to a brief survey of the notions of analysis and synthesis in some Neoplatonists. I show that both the mathematical and the philosophical versions of the analytic/synthetic method became associated with several new contexts.

Keywords: Analysis, synthesis, geometry, negative theology.

Resumen: En este artículo se investigan los orígenes griegos y latinos de la máxima escolástica que dicta que lo primero en la composición (*sunthesis*) es lo último en la resolución (*analysis*). Estudio a la tradición que sostuvo que la síntesis procede de lo anterior a lo posterior, mientras que el análisis procede de lo posterior a lo anterior. En primer lugar, busco los orígenes de estas nociones en las matemáticas griegas. Posteriormente me ocupo de autores del platonismo medio, quienes identificaron el análisis con la *diairesis* platónica. Paso después a un breve estudio de las nociones de análisis y síntesis en algunos neoplatónicos. Muestro que tanto la versión matemática como la filosófica del método analítico/sintético se asoció con varios contextos nuevos.

Palabras clave: Análisis, síntesis, geometría, teología negativa.

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0. INTRODUCTION

This article is intended as a contribution to the history of the philosophical methods of analysis and synthesis, notions that play an important role in the thought of Thomas Aquinas and other Scholastic thinkers. Since the function of these notions in Aquinas' thought has been well studied elsewhere, I restrict myself to a brief reminder of this point. Yet since it has been claimed that one of Aquinas' main sources is a passage from Chalcidius, who in turn derived these notions more or less directly from Aristotle, I study the Chalcidius passage in some detail, before attempting to show that Chalcidius is merely a fairly late representative of a long tradition that goes back, in all likelihood, to the Platonic Academy.

Rather than breaking new ground, then, the paper has the ambition of providing a survey of a centuries-long development in philosophy and mathematics. My approach will be primarily descriptive, rather than critical. I believe there is room, in the history of philosophy, for such an approach, since it is surely necessary to gain as complete an understanding as possible of a problem before proceeding to criticize the solutions proposed for it by an individual philosopher. This entails that I shall have to give short shrift to a number of interesting questions, each of which could, and perhaps should, be the subject of a separate article.¹ In general, I will largely be taking the authors I study at face-value, assuming, as I believe the principle of charity allows me to do, that they more or less mean what they say. Deconstructionism can be an illuminating approach, but it is not the only one possible, and it is not the one I have chosen to apply in this paper.

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1. These include, in the case of Galen, the question of the extent to which his recourse to the methodology of analysis and synthesis might be largely rhetorical, as part of his "mathematical ideology" and "self-promotional agenda". Similarly, I will not go into the question of the extent to which Alexander of Aphrodisias' recourse to these notions is, or is not, faithful to the thought of Aristotle as expressed in *Metaphysics* Λ. These questions, raised by an anonymous reviewer, are important, but would each require at least an article-length study for adequate treatment.

There is, however, a sense in which this paper will attempt to delve beneath the surface meaning of the texts it studies. It will emerge that the methodology of analysis and synthesis has a very long history, reaching back beyond the Platonic Academy to early Greek mathematics. When someone like Chalcidius, writing more than seven centuries after Plato, comes to appeal to these notions, it may well be that he does not fully understand them, unaware as he must have been of all of their historical vicissitudes. This is true *a fortiori* of Albertus Magnus, Aquinas and his fellow Scholastics, who wrote another eight hundred years after Chalcidius. What they lacked in historical knowledge, however, they often made up for by what Pierre Hadot has identified² as “creative mistakes”: the creative re-interpretation, sometimes based on misunderstandings and inaccurate translations, of ancient philosophical themes and problems in terms of contemporary concerns. As Hadot has shown, this process often leads to fruitful and interesting developments on the history of philosophy.

Before we can identify the presence and absence of such creative mistakes, however, and hence evaluate the “originality” of a given author with regard to the tradition and genre constraints he worked in, we must try to re-trace the actual historical origins of the complex of ideas in question. This was the function of the discipline once known as the History of Ideas, and although it is no longer very fashionable, this is the methodology I have chosen to employ in the present study. On the basis of the material gathered here, others will, I hope, be able to apply a wide variety of methodologies to the study of analysis and synthesis in one or more of the philosophical or mathematical thinkers I have briefly evoked in what follows.

2. P. HADOT, *Philosophy as a way of life. Spiritual Exercises from Socrates to Foucault*, edited with an introduction by Arnold I. Davidson, translated by Michael Chase (Oxford, Blackwell, 1995) 71-77.

1. ANALYSIS AND SYNTHESIS IN THOMAS AQUINAS

A recurrent theme in the work of Albertus Magnus³ and Thomas Aquinas is the formula stated in the title of this contribution: that which is first in composition (equivalent, as we shall see, to the Greek *sunthesis*) is last in resolution (Greek *analysis*).

Aquinas formulates the doctrine explicitly several times in these terms: what is first in composition is last in resolution,⁴ and conversely what is first in resolution is last in composition.⁵ The *via compositionis* leads from the prior to the posterior, while the *via resolutionis* leads from the posterior to the prior;⁶ from what is compound to what is simple.⁷ The *modus resolutivus* or *via resolutionis* resolves compounds into its parts,⁸ whether these be simple principles⁹ or individuals,¹⁰ or to intellectual substance,¹¹ otherwise known as the transphysicals,¹² which are indivisible and the cause of things;¹³ whereas the *modus compositivus* proceeds from the sim-

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3. See, among many examples, ALBERTUS MAGNUS, *De causis et processu universitatis a prima causa*, 2.1.17, p. 81, 12-14 Fauser: Quod autem ultimum est in resolutione primum necesse est esse secundum viam compositionis. Albert knows of analysis from the compound to the simple, or from effect to cause "according to the order of formal causes", or from the posterior to the prior "according to nature and intellect", and finally from particular to universal (*ibid.* p. 81, 7-11).
 4. *In Metaph.* 5, lect. 4, no. 799, p. 219: quod est enim ultimum in resolutione, oportet esse primum in compositione; *In II Sententiarum*, dist. 22, quaest. 1, art. 1, resp. ad argum. 2; *In III Sententiarum*, dist. 2, quaest. 2, art. 3, quaestiuncula 2, arg. 1; *In De caus.* 1, 5, 10. Cf. already Clarenbald of Arras, *Tractatus super librum Genesis*, sect. 21, p. 235 ed. Häring: quicquid enim est ultimum in dissolutione, primum est in compositione.
 5. *Quaestiones disputatae de uirtutibus, de spe quaestio unica*, art. 3, respons. ad arg. 11, p. 809, 1.
 6. *In De cael.* 2, lect. 4, 4, 2, p. 136.
 7. *In Metaph.* 2, lect. 1, 278, p. 81.
 8. *In Phys.*, 3, 1, no 3, 25, p. 140.
 9. *Sententia libri Ethicorum*, 1, 2, no 35, 9 p. 10 Marietti = 11 Leonina.
 10. *In Polit.*, 1, 1, no 16, 27, p. 7 Marietti.
 11. *De substantiis separatis* 5, 62, 22 p. D 48 ed. Leonina.
 12. *In Metaph.* 12, 2, Prooemium.
 13. *In Metaph.* 10, lect. 2, no. 1952, 11, p. 467.

ple to the compound, or from cause to effect.¹⁴ The *modus compositivus* is to be used in the practical sciences, the *modus resolutivus* in speculative sciences.¹⁵

While there are Aristotelian texts that hint at this methodology of analysis and synthesis,¹⁶ it seems to me that here, as so often, Thomas Aquinas is dependent not so much on the text of the Stagirite as on a long tradition of Aristotelian commentators,¹⁷ writing in Greek, Latin, and Arabic. I will only have space to hint at the Arabo-Islamic sources in the present contribution, and the role of analysis and synthesis in Thomas Aquinas is best left to professional Aquinas scholars, who have already produced excellent studies on the subject.¹⁸ I will therefore limit myself to sketching the broad outlines of the history of the notions of analysis and synthesis in Greco-Roman thought down to the 6th century CE.

14. *Summa Theologiae prima secundae*, quaest. 14, art. 5, arg. 1; corpus, line 2.

15. *Sententia libri Ethicorum*, 1, 2, no 35, 9 p. 10 Marietti = 11 Leonina.

16. L. P. SCHRENK, *Proof and discovery in Aristotle and the later Greek tradition: a prolegomenon to a study of analysis and synthesis*, (ed.), *Aristotle in Late Antiquity*, "Studies in Philosophy and the History of Philosophy" 27 (1994) 92-108. Schrenk cites *Anal. post.* 78a61-3, *Metaph.* Theta 9, 1051a21-33; *EE* 1227b28-33; and, perhaps most relevantly, *NE* 3.5, 1112b18-20, where Aristotle states, referring to geometrical proofs, that what is last in the order of analysis seems to be first in the order of becoming (ἕως ἄν ἔλθωσιν ἐπὶ τὸ πρῶτον αἴτιον, ὃ ἐν τῇ εὐρέσει ἔσχατον ἐστὶν (...) τὸ ἔσχατον ἐν τῇ ἀναλύσει πρῶτον εἶναι ἐν τῇ γενέσει). I agree with Schrenk, however (1994, 100), that while Aristotle probably knew of a geometrical tradition making use of the analysis/synthesis methodology, he has "not yet adapted it as an explicit philosophical procedure". Sorabji does not hesitate to affirm that Aristotle "borrows from geometry the idea of analysis". R. SORABJI, *The philosophy of the commentators, 200-600 AD. 400 years of transition: a sourcebook*, vol. 3, 1, *Psychology (with ethics and religion)*; 2, *Physics*; 3, *Logic and metaphysics*. (Duckworth, London, 2004) 268.

17. E. C. SWEENEY, *Three Notions of Resolutio and the Structure of Reasoning in Aquinas*, "The Thomist" 58 (1994) 205.

18. Cf. *In primis* E.C. SWEENEY, *op. cit.*, with further literature, to which one may add J. A. AERTSEN, *What is first and most fundamental? The beginnings of transcendental philosophy*, in J. A. AERTSEN, A. SPEER (eds.), *Was ist Philosophie im Mittelalter?* (De Gruyter, Berlin /New York, 1998) (*Miscellanea Mediaevalia* 26) 177-192; J. F. WIPPEL, *Metaphysical Themes in Thomas Aquinas*, ch. III, in *Studies in Philosophy and the History of Philosophy*, vol. 10 (The Catholic University of America Press, Washington, D.C., 1984) 62ff.

2. ANALYSIS AND SYNTHESIS IN CHALCIDIOUS¹⁹

The earliest Latin testimony to the use of the analysis-synthesis scheme I have been able to find occurs in Calcidius' *Commentary on the Timaeus*, written sometime in the fourth or early fifth century. In the midst of his mini-treatise on matter, Calcidius writes as follows:²⁰

The proof of proposed problems²¹ is therefore twofold: one which confirms posterior things from more ancient ones, which is proper to the syllogism—indeed, assumptions,²² which are called elements,²³ precede the conclusion in

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19. Cf. E.C. SWEENEY, *op. cit.*, 207ff., for whom this passage from Calcidius is one of Thomas' three main sources for Aquinas' notion of analysis, the others being Neoplatonism (Proclus, Ps-Dionysius and Eriugena), and the Greek geometrical tradition. This may be, but the author's failure to distinguish between analysis, abstraction, and division, and her neglect of the entire Old Academic and Middle Platonic tradition lead her to the wholly erroneous notion that Calcidius is directly dependent on Aristotle (cf. p. 211, where she goes so far as to speak of Calcidius' "literal interpretation" of Aristotle's notion of analysis). As we shall see, if the method in question can be said to be "Aristotelian" at all, it is so only because Aristotle, like Plato and the other Academic founders of the analysis-synthesis method, took their inspiration from the common source of contemporary geometry.
 20. Calcidius, *In Tim.*, paragraph 302, Vol. I, p. 530, 5-9. B. BAKHOUCHE, *Calcidius, Commentaire au Timée de Platon*, édition critique et traduction française, avec la collaboration de LUC BRISSON (trad.), *Histoire des Doctrines de l'Antiquité Classique*, XLII (Librairie Philosophique J. Vrin, Paris, 2011). Bakhouché: Est igitur propositarum quaestionum duplex probatio, altera quae ex antiquioribus posteriora confirmat, quod est proprium syllogismi - praecedunt quippe ordine acceptiones, quae elementa uocantur, conclusionem -, altera item, quae <ex> posterioribus ad praecedentium indaginem gradatim peruenit, quod genus probationis resolutio dicitur.
 21. With Calcidius' *propositarum quaestionum* cf. PAPPUS, *Synagogê*, bk. VII, p. 634, 6-7: *proteinomena problêmata*.
 22. Calcidius' *acceptio* no doubt corresponds to the Greek *lêpsis*, "assumption" (LSJ s.v. iii). Cf. IBN AL-TAYYIB (ob. 1043), discussing synthesis in his *Commentary on Galen's Ars medica*, British Library Arund. 52 = Or. 57, fol. 5^r, quoted in the Italian translation (I have not seen the Arabic) by I. GAROFALO, *Il commento di Abû l-Farâj ibn at-Tayyib all'Ars medica di Galeno*, in N. PALMIERI (ed.), *L'Ars Medica (Tegni) de Galien: lectures antiques et médiévales. Actes de la «Journée d'étude internationale» (Saint-Étienne, 26 juin 2006)*, (Publications de l'Université de Saint-Étienne, Saint-Étienne, 2008) (Centre Jean Palerne, Mémoires XXXII) 116. Garofalo says: "il sillogismo non è nulla più dell' assunzione [*lêpsis*! - MC] di premesse semplici (...) e da esse viene una conclusione".
 23. On "taking by means of elements" (*lêpsis dia stoikheîôn*) see, for instance, ALEXANDER OF APHRODISIAS, *In anal. pr.*, p. 379, 21 ff. Wallies.

order—while the other arrives gradually at the investigation of preceding things from posterior things,²⁴ and this kind of proof is called analysis.

Calcidius here identifies two methods of enquiry, proof, or argumentation. One, which he does not name, but likens to syllogistic reasoning, proceeds from the prior to the posterior—we will see shortly that it corresponds to the method which the Greeks called *synthesis*—; the other, called *resolutio* (= Greek *analysis*), proceeds from the posterior to the prior. Since his current goal is the investigation of matter, Calcidius announces that he will use the method of *resolutio*.

The notions of prior and posterior are, however, relative, so Calcidius proceeds to specify what he means by them. After distinguishing between the eternal intelligibles and sensible things, which have a beginning in time, he points out that sensible things are primary from our viewpoint but secondary according to their nature, while intelligibles are secondary from our perspective, but primary in reality.²⁵ Thus, the procedure of starting out from sensible things—secondary in reality but primary *ad nos*—and ascending to the intelligibles—secondary *ad nos* but primary in reality—is that of analysis (*resolvere dicitur quaestionem*).²⁶

In Ch. 303 of his commentary, Calcidius sets out to use the process of analysis, and we see right away that that it consists in a process of abstraction. Our bodies and the world in general possess such qualities as visibility, warmth, moisture, etc., which allow us to conclude to the existence of the four elements, as well as to the existence of something that holds together these aggregates of forms

24. Cf. IBN AL-ṬAYYIB, *Commentary on Galen's Ars medica*, fol. 5^v, p. 117 Garofalo: "... l'analisi avviene cominciando dai principi prossimi della cosa finché arriviamo ai principi remoti".

25. The inverse relation between what is knowable *per se* and knowable *ad nos* is, of course, Aristotelian in origin: cf. *APo* I, 2, 72a1-5; *Physics* I, 1, 184a16-25; *Metaph.* D 11, 1018b30-37; Z 10, 1035b12; M 2, 1077b1.

26. Similarly, IBN AL-ṬAYYIB, *op. cit.*, fol. 4^v, trans. p. 115 Garofalo, notes that the path of analysis is contrary to that of nature, in that nature begins with what is simple and ends with what is compound, while the path of analysis "comincia da ciò che è più manifesto verso il più oscuro".

and qualities. We are to think away all these forms and qualities, and what's left over will be matter.

In Chapter 304, Calcidius comes to the other method, which we now learn is called *compositio* (Greek *synthesis*): it is inverse to, and follows upon, analysis. In this method, we put back together the things we had mentally separated by the process of analysis, viz. the genera, qualities, and figures.²⁷ At the end of this process, Calcidius announces, we have reached the stage of discovering the divine *opifex*, who regulated, ordered and adorned the formation of the cosmos, following his thoughts, which are the divine Ideas,²⁸ *qua* models of all natural things. Thus, whereas the method of analysis has revealed matter, the passive principle, synthesis has revealed God, the active principle whose mind is equivalent to Providence.

Calcidius thus presents a methodology in which one starts out from the sensible world around us and, by using analysis, envisaged as a process of abstraction, arrives at bare matter. Thanks to a combination of Platonic and Aristotelian doctrines, this can also be considered as a progress from what *seems to us* to be primary or prior (the entities of sensible world) to what is *truly* or *naturally* prior, viz. the intelligible. This method of analysis (*resolutio*) is immediately followed by the method of synthesis (*compositio*), in which, starting out from what is less familiar to us but ontologically prior (in this case, *matter*), we put back together what we had separated by abstraction in the process of analysis, and arrive at knowledge of God *qua* creator of the sensible world.

Calcidius' account seems somewhat confused. On the one hand, we have a notion of analysis as equivalent to a process of abstraction; on the other, a sense in which analysis is paired with synthesis as part of a methodology which, as we shall see, is mathematical in origin. Part of the goal of this contribution will be, by

27. According to Calcidius, synthesis is to be carried out as follows: order implies harmony, harmony analogy, analogy reason or proportion (Greek *logos*), and reason/proportion implies providence. Providence, in turn, implies intellect, and intellect mind.

28. That the Platonic Forms or Ideas are the thoughts of God is in origin a Middle Platonic doctrine, but we find in in such Neoplatonists as AMMONIUS, *In Isag.*, 41,21-42, 7.

indulging in some *Quellenforschung*, to see whether these two notions are ultimately reconcilable.

3. ANALYSIS AND SYNTHESIS IN GREEK MATHEMATICS: FROM THE ZÊTOUMENON TO THE HOMOLOGOUMENON AND BACK

The method of analysis and synthesis is well attested in Greek mathematics and geometry, whence it had a huge influence on the development of such modern mathematical pioneers as Vieta, Galileo and Newton. Its actual occurrence in ancient theoretical discussions,²⁹ as opposed to its tacit use, is, however, later than one might have supposed, at least insofar as the surviving texts allow us to determine.

3.1. *The scholium to Euclid*³⁰

The only mention we find of the method of analysis and synthesis in Euclid's *Elements* (c 300 BCE), for instance, comes in the form of a scholium to Book XIII, in which it is claimed that in the book's first five theorems, synthesis results from analysis by means of conversion:³¹

What is analysis and what is synthesis.

Analysis is the assumption (Greek *lêpsis*) of what is sought (*tou*

29. I add the qualification "in theoretical discussions" because, as Rashed has pointed out, texts such as Archimedes' *On the Sphere and the Cylinder* 2.4-6, Apollonius' *Introduction to Conics* 4.16, and Diophantes' *Arithmetic* seem to tacitly presuppose familiarity with the analysis/synthesis scheme. R. RASHED, *L'analyse et la synthèse selon Ibn al-Haytham*, in id. (ed.), *Mathématiques et philosophie de l'Antiquité à l'âge classique. Hommage à Jules Vuillemin*, (CNRS, Paris, 1991) 131-149. On synthesis in Apollonius, cf. J. MANSFELD, *Prolegomena mathematica: from Apollonius of Perga to late Neoplatonism: with an appendix on Pappus and the history of Platonism* (Brill, Leiden, 1998), 9-10.

30. M. PANZA, *Classical sources for the concepts of analysis and synthesis*, in M. OTTE, M. PANZA (eds.), *Analysis and synthesis in mathematics*, "Boston Studies in the Philosophy of Science" 196 (Kluwer, Dordrecht, 1997) 386-389.

31. EUCLID, *Elementa* (demonstrationes alterae, lib. 11-13), scholium to XIII, 1-5, Vol. 4, 364f. Heiberg-Menge.

zêtoumenou) as admitted (*hôs homologoumenou*) <to arrive>³² by means of the consequences at something admitted to be true. Synthesis is the assumption of something admitted <to arrive> by means of the consequences to the complete understanding of what is sought.

In this text, which is almost certainly a late addition to the text of Euclid, perhaps due to Heron of Alexandria, the two methods of analysis and synthesis are envisaged as the inverse of one another. In analysis, one assumes what is under investigation (*to zêtoumenon*) and proceeds to something generally admitted to be true (*to homologoumenon*). When we come to the description of synthesis, however, we encounter a serious textual problem.³³ If we follow the text printed in Heiberg's critical edition of Euclid, the goal of the process of synthesis is identical to the goal of analysis: in both methods, one arrives at something admitted to be true (*epi ti alêthês homologoumenon*). Yet if, as we probably should,³⁴ we adopt the reading of several other authoritative manuscripts, the goal of synthesis is no less than the complete understanding of what is sought (*tên tou zêtoumenou katalêxin êtoi katalêpsin*).³⁵ On this latter understanding of the text, synthesis becomes, instead of a superfluous process leading to the

32. I follow Vitrac in supplying these words. B. VITRAC, *Euclide d'Alexandrie, Les Éléments, traduits du texte de Heiberg*, I: Introduction générale; Livres I-IV: géométrie plane; 2, Livres V-VI: Proportions et similitude; Livres VII-IX: Arithmétique; 3, Livre X: Grandeurs commensurables et incommensurables, classification des lignes irrationnelles; 4, Livre XI-XIII: Géométrie des solides, (Pr. Universitaires de France, Paris, 1990-2001) vol. 4, 392.

33. Ms. P (= Codex Vaticanus, Gr. 190, 10th cent.), the text printed by Heiberg, reads *epi ti alêthês homologoumenon* (something admitted to be true). I follow Vitrac in adopting the reading *tên tou zêtoumenou katalêxin êtoi katalêpsin*, given by mss. B (Codex Bodleianus, D'Orville X, copied in 888); V (Codex Vindobonensis, philo. Gr. 103, probably 12th cent.); b (Codex Bononiensis, Communal Library, no 18-19, 11th cent.); and q (Codex Parisinus, Gr. 2344, 12th cent.).

34. B. VITRAC, *op. cit.*, 392, argues convincingly that the version contained in P is a scribal error resulting from homoeoteleuton. Cf. already A.-J. FESTUGIÈRE, *La révélation d'Hermès Trismégiste. I: L'Astrologie et les sciences occultes. II: Le dieu cosmique. III: Les doctrines de l'âme. IV: Le dieu inconnu et la gnose*, (Éditions J. Gabalda, Paris, 1954) vol. 4, 120 n. 2.

35. The words *katalêxis* and *katalêpsis* are virtual synonyms; in fact, the words *êtoi katalêpsin* may constitute a marginal gloss.

same result as the method of analysis, a method by which we confirm, amplify, and complete the results obtained through analysis.

3.2 Pappus of Alexandria³⁶

Some six hundred years after Euclid, a more detailed exposition of analysis and synthesis was provided by Pappus of Alexandria, at the beginning of the seventh book of his *Mathematical collection* (written c. 340CE).³⁷

Now, analysis is the path from what is sought³⁸ taken as admitted, through the subsequent consequences, to something admitted by synthesis. In analysis, supposing what is being sought to have come to be (*hôs gegonos*), we investigate out of what this comes about, and again what precedes this, until, backtracking³⁹ in this way, we come upon something already known or which has the rank of a principle. We call this approach analysis, as a reverse solution.⁴⁰ In synthesis, we reverse our steps and suppose to have already come to be what was grasped as last <step> in analysis, and arranging what were previously the antecedents in their natural order as consequences, and adding them succes-

36. This text has been often discussed by historians of philosophy and science. Cf. A. JONES, *Pappus of Alexandria: Book 7 of the Collection*, vol. 8 (Springer, New York, 1986); J. HINTIKKA & U. REMES, *The Method of Analysis* (Reidel, Dordrecht, 1974), 8; M. PANZA, *op. cit.*, 383-385.

37. PAPPUS, *Collection mathematica*, vol. II, 31-44. Hultsch, trans. based on J. KLEIN, *Greek mathematical thought and the origin of algebra*, E. BRANN (trad.) (Dover, New York, 1992) 260.

38. *Apo tou zêtoumenou*. Cf. PLATO, *Meno* 79d.

39. *Anapodizontes*. Stobaeus (*Anthologium* I, 10, 12) quotes the Pythagorean Mnesarchus of Samos on the nature of the decad: "Everyone counts as far as ten, and when they reach it they once again return to the monad" (μέχρι γὰρ τῶν δέκα πάντες ἀριθμοῦσιν, ἐφ' ᾧ ἐλθόντες πάλιν ἀναποδίζουσιν ἐπὶ τὴν μονάδα).

40. An attempt at etymological explanation: *analusis* is supposed to come from *anapalin* + *lusis*, a "reverse solution". Cf. ELIAS, *In Isag.*, 37, 21-3, discussed below; J. MANSFELD, *op. cit.*, 123. Cf. IBN AL-ṬAYYIB, *On Galen's Medical Art*, fol. 4^v, 115. Garofalo: "...questa via è chiamata soluzione [*lysis*] poiché l'intelletto scoglie in essa il composto nei suoi principi, e all'inverso [*ana-*] poiché l'intelletto percorre in essa il contrario della via naturale".

sively to each other, we arrive at the goal of constructing what was sought, and this is what we call synthesis.

As we can see, Pappus reproduces, in more complete and explicit form, the basic scheme we found the scholium to Euclid: analysis is the path from the *zêtoumenon* (what is sought) to the *homologoumenon* (what is agreed upon, accepted, or admitted). Assuming the conclusion of our reasoning, we backtrack to see what its presuppositions are, and we continue in this way until we reach some proposition already known, or some indemonstrable principle. More explicitly than the scholium, Pappus specifies that this last stage of analysis is the first stage of synthesis: in this latter process, we start out from the generally-admitted or indemonstrable principle we have reached by analysis, and go over the same series of steps we had covered in the process of analysis, but in reverse order, so that what were antecedents or presuppositions in analysis become consequences (Greek *hepomena*) in synthesis. By adding all these steps together at the end of our process of synthesis, we finally come to the *zêtoumenon*.⁴¹

3.3 Heron of Alexandria and the Arabo-Islamic geometrical tradition⁴²

One of the missing links between Euclid and Pappus is provided by a passage from Heron of Alexandria's (c. 10-70 CE) commentary on Euclid's *Elements*. Fragments of this work, lost in the original Greek, are preserved in the Arabic commentary on Euclid by the Persian mathematician Abu 'l-'Abbās al-Faḍl b. Ḥātim al-Nayrīzī

41. In what follows, Pappus then goes on to distinguish between two kinds of analysis: theoretical, used in discovering the proof of a theorem, and problematic, used in the solution or construction of a problem. In the former kind, synthesis, or reversal of the analysis, constitutes an immediate demonstration of what was being sought; whereas in the latter kind, synthesis consists first in a geometrical construction or porism, which is then followed by the demonstration.

42. On analysis and synthesis in Arabo-Islamic philosophy, see also A. HASNAWI, *Topic and analysis: the Arabic tradition*, in R. W. SHARPLES (ed.), *Whose Aristotle? Whose Aristotelianism?* (Ashgate, Aldershot etc., 2001) 29-32; 39.

(c. 900CE). Translated into Latin by Gerard of Cremona, this commentary by al-Nayrīzī (known to the Latins as Anaritius) was well known to the Scholastic tradition, particularly to Albertus Magnus.⁴³ Al-Nayrīzī quotes Hero as follows:⁴⁴

And furthermore, if we proceed by positing just one line, we can present two methods of demonstration (*al-burhān*), one of the two of which is the method of analysis (*ṭarīq al-tahlīl*), and the other the method of synthesis (*ṭarīq al-tarkīb*).⁴⁵ As for analysis, it is when some problem or other is proposed to us, and we say, “We suppose that what is sought⁴⁶ is extant”.⁴⁷ Then we resolve it to something whose demonstration has preceded. Then when it has become clear, we say, “What is sought has been found by analysis”. And as for synthesis, that is when one begins with known things;⁴⁸ then one combines them until what is sought is found, and thus what is sought has been made clear by synthesis.

While this text provides little additional information above and beyond that of the Euclid scholium and Pappus, it does help us to establish terminology. The standard correspondences are as follows:

43. I discuss Albertus Magnus’ use of al-Nayrīzī in M. CHASE, In press, *Albert le Grand sur la dérivation des formes géométriques: Un témoignage de l’influence de Simplicius par le biais des Arabes?*, to be published in the Acts of the Conference on “Damascius et le parcours syrien du néoplatonisme”.

44. A. LO BELLO, *The commentary of al-Nayrizi on Books II-IV of Euclid’s Elements of Geometry, with a translation of that portion of Book I missing from MS Leiden Or. 399.1 but present in the newly discovered Qom manuscript edited by Rudiger Arnzen*, (Brill, Leiden, 2009). AL-NAYRĪZĪ, 8, 1-8 Besthorn/Heiberg, translation Lo Bello modified.

45. Cf. Gerard of Cremona: *modi, quorum unus est modus, qui attenditur secundum dissolutionem*, alter vero modus, qui consideratur *secundum compositionem*.

46. Arabic *al-maṭlūb* = Latin *res quesita* = Greek *to zētoumenon*.

47. Ar. *mawjūd*. Lo Bello translates “true”, but *mawjūd* corresponds more closely to the Greek *on* or *gegonos*, “extant” or “having come to be”, as in Pappus, although the Arabic can also mean “found”.

48. Ar. *bi-ašyā’ ma’rūfa*.

Greek	Arabic	Latin (Gerard)
<i>analysis</i>	<i>al-tahlīl</i>	<i>dissolutio</i>
<i>synthesis</i>	<i>al-tarkīb</i>	<i>compositio</i>

As has been pointed out by Roshdi Rashed, the methodology of analysis and synthesis, known to the Arabic world thanks to Hero and other sources, played an increasingly important role in Arabic-language mathematics and geometry beginning with the tenth century, primarily as a result of developments within Arabo-Islamic algebra and geometry. Thus, the mathematician Ibrāhīm ibn Sinān (909-946) wrote a work entitled *Fī ṭarīq al-tahlīl wa-l-tarkīb fī-l-masā'il al-handasiyya* ("On the method of analysis and synthesis in geometrical problems").⁴⁹ Ibn Sinān's work was elaborated upon by such scholars as al-Sijzī (last third of the 10th century),⁵⁰ and especially Ibn al-Hayṭam⁵¹ (died after 1040). For the latter, the art of analysis (*ṣinā'a al-tahlīl*) provides the method for obtaining demonstrative syllogisms, resulting in an *ars demonstrandi*. More generally, the continuation of the process until one arrives at "known things" (Ar. *bi-ašyā' ma'rūfa*) was to become the key point in the later Arabic development of the analysis/synthesis scheme. Ibn al-Hayṭam, for instance, interprets the "known things" as properties which remain invariant even when a geometrical figure is transformed: only when analysis has discovered these invariant properties can demonstration be carried out, thanks to the method of synthesis.⁵²

49. R. RASHED, H. BELLOSTA, *Ibrāhīm ibn Sinān, logique et géométrie au Xe siècle* (E.J. Brill, Leiden, 2000).

50. R. RASHED, *Les mathématiques infinitésimales du IXe au XIe siècle. 4, Ibn al-Haytham, Méthodes géométriques, transformations ponctuelles et philosophie des mathématiques*, (Al-Furqān, London, 2002) and *Filosofia della matematica*, in *Storia della Scienza. 3, La civiltà islamica*, in R. RASHED (coord.), R. MORELON, U. WEISSER (col.) (Istituto della Enciclopedia Italiana, Roma, 2002) 483-498.

51. R. RASHED, *op.cit.* 1991; *op. cit.* 2002, 157ff.

52. This is not the place to embark upon a description of the fascinating development of the methods of analysis and synthesis in Arabo-Islamic geometry and philosophy, for which I refer the reader to the works of Roshdi Rashed and colleagues.

4. THE MIDDLE PLATONISTS: *ANALYSIS* = PLATONIC *DIAIRESIS*

Now that we have made a brief excursion into the world of Greek and Arabo-Islamic mathematics, we are in a better position to understand the development of the notions of analysis and synthesis in that age of Greek philosophy often referred to as the Middle Platonist period. This in turn will provide us with a better grasp of the sources of Chalcidius and hence, of at least one of the main sources of this complex of ideas in Thomas Aquinas.

4.1 *Galen (c. 129-c. 200 CE):*

from the first to the many, and back again

Turning from mathematics to medicine, we find that the great physician and philosopher Galen⁵³ was quite familiar with the notions of analysis and synthesis, as we learn from the introductory paragraphs of his *On the medical art*:⁵⁴

Three are all the teachings that proceed in an orderly way. The first is that which comes about from the concept of the end by means of analysis (*ek tês tou telous ennoias kat'analusin*). The second is that <which takes place> from the synthesis of the things found by analysis. The third is <that which takes place> from the resolution (*dialusis*) of a definition, which we are now undertaking (...)

53. For Galen's views on the method of *diairesis*, see J. MANSFELD, *Heresiography in Context: Hippolytus' Elenchos as a Source for Greek Philosophy* (Brill, Leiden, 1992) 330; for analysis in Galen see M. HAVRDA, *Galenus Christianus? The doctrine of demonstration in Stromata VIII and the question of its source*, "Vigiliae Christianae" 65 (2011) 366; R. CHIARADONNA, *Scienza e contingenza in Galeno*, in S. PERFETTI (ed.), *Conoscenza e contingenza nella tradizione aristotelica medievale*, (ETS, Pisa, 2008) 40, with further references.

54. GALEN, *On the Medical art*, I, 274, 1-275 1 Boudon = vol. I, pp. 305-306 Kühn. This work had a huge influence: it was translated into Syriac by Sergius of Resh'ainā, into Arabic by Hunain ibn Ishāq, into Hebrew by Samuel ibn Tibbon and into Latin by Gerard of Cremona, among others. Cf. N. PALMIERI, *Survivance d'une lecture alexandrine de l'Ars medica en latin et en arabe*, "AHDLM" 60 (2003) 57-102, with further references concerning the posterity of this text down to the Renaissance.

In Galen's time, then, the methods of analysis and synthesis already constituted, together with the method of definition, two of the most prominent methodologies of medical research, although Galen himself preferred the third, definitional approach. Later on, in the Latin-speaking world, Galen's division into analysis, synthesis and analysis of definition was to be conflated with a similar division transmitted by Boethius:⁵⁵

And of this [sc. logic], the division is, in one sense, triple, for every power of the logical discipline either defines something, or divides it, or collects it (*omnis namque uis logicae disciplinae aut definit aliquid, aut partitur, aut colligit*).

Here, Boethius' *definit* corresponds, in all likelihood, to the Greek *horismos*; his *partitur* to the Greek *diairesis* or *analysis*, and his *colligit* to the Greek *sunagôgê*.

Yet Galen was also aware of a more purely philosophical method of analysis and synthesis, as we learn from his *De placitis Hippocratis et Platonis*:⁵⁶

55. BOETHIUS, *In Topica Ciceronis Commentaria*, PL 64, col. 1045B. Equally influential on Latin posterity was Boethius' alternative division (*ibid.*, 1045C) of philosophy as *ratio disserendi* into a *pars inueniendi* and a *pars iudicandi*.

56. GALEN, *De placitis Hippocratis et Platonis*, 9.5.9-14, vol. II, 566, 4-26 De Lacy: ...τά γε παραδείγματα κατὰ ταῦτόν ἐν τοῖς παραβαλλομένοις ἐκέλευσεν ἡμᾶς ὁ Πλάτων ποιῆσθαι. γειτνιά δέ πως τούτῳ τῷ σκέμματι καὶ τὸ κατὰ τὴν διαιρετικὴν ὀνομαζομένην μέθοδον ἧς τὴν μὲν γυμνασίαν ὁ Πλάτων ἐν Σοφιστῇ καὶ Πολιτικῷ πεποιήται, τὴν δὲ ἐξ αὐτῆς χρεῖαν ἐπέδειξεν οὐκ ἐν τούτοις μόνον, ἀλλὰ σαφέστατα μὲν ἅμα καὶ τελεώτατα κατὰ τε Φίληβον καὶ Φαῖδρον, οὐ μὴν <ἀλλὰ> καὶ κατὰ τὴν Πολιτείαν τε καὶ ἄλλ' ἅττα τῶν συγγραμμάτων. ἐν μὲν οὖν τῷ Σοφιστῇ καὶ τῷ Πολιτικῷ διδάσκει πῶς ἂν τις ἀντὶ τῆς προσηγορίας ἐρμηνεύει λόγῳ σαφεῖ τε ἅμα καὶ συντόμῳ τὸ σημαινόμενον αὐτῆς, ὅντινα λόγον ὀρισμὸν τε καὶ ὄρον ἐξαιρέτως ἐκάλεσαν οἱ μετὰ Πλάτωνα. ἐν δὲ τῷ Φιλήβῳ καὶ τῷ Φαῖδρῳ δείκνυσιν εἰς τεχνῶν σύστασιν ἀναγκαιοτάτην εἶναι τὴν διαιρετικὴν καὶ συνθετικὴν θεωρίαν γεγυμνάσθαι τε κελεύει διττῶς κατ' αὐτὴν ἀπὸ μὲν τοῦ πρώτου καὶ γενικωτάτου καταβαίνοντας ἐπὶ τὰ μηκέτι τομὴν δεχόμενα διὰ τῶν ἐν τῷ μεταξὺ διαφορῶν, δι' ὧν καὶ τοὺς ὀρισμοὺς τῶν εἰδῶν ἐδεδείχθαι συνισταμένους ἐν Σοφιστῇ καὶ Πολιτικῷ, ἔμπαν δ' ἀπὸ τῶν εἰδικωτάτων πολλῶν ὄντων ἀναβαίνοντας ἐπὶ τὸ πρῶτον γένος κατὰ σύνθεσιν· ὁδὸν μὲν γὰρ εἶναι μίαν ἀμφοῖν, ὁδοιπορίαν δὲ διττὴν ἀπὸ θατέρου τῶν πρώτων ἐπὶ θάτερον ἐναλλάξ ἰόντι.

... Plato recommended that we carry out our examples in accordance with what is the same in things being compared. A kind of neighbor to this scheme is the one by means of the so-called divisive method (*kata tēn diairetikēn onomazomenēn methodon*). Plato carried out the practice of it in the *Sophist* and the *Statesman*, but he showed its usefulness not only in these, but in a way that is simultaneously most clear and most complete, according to the *Philebus*⁵⁷ and the *Phaedrus*, but also in the *Republic* and in other works. In the *Sophist* and the *Statesman*, then, he teaches how, instead of the appellation (*prosêgoria*), one might explain its meaning in a discourse that is both clear and concise,⁵⁸ which discourse Plato's successors specially called a definition and a term (*horismos te kai horon*). In the *Philebus* and the *Phaedrus*, he shows that for the constitution of the technical arts, the theory of division and synthesis is most necessary, and he recommends that one should be trained in it in two ways: *by descending from what is first and most generic to the things that are no longer susceptible of division, by means of the intermediary differences*, by which he had shown in the *Sophist* and the *Statesman* that definitions are constituted, *and contrariwise, ascending from the many most specific things to the first genus, via synthesis*. For the path is the same for both, but the journey is twofold, going alternately from one of the first things to the other.

Galen then goes on to quote *Phaedrus* 265c-e. He thus knows of a philosophical method of analysis, here equated with 'division' (*di-*

57. For the *Philebus* cf., for instance, DAMASCIUS, *In Phileb.*, 52-56, with the notes to the Budé edition by VAN RIEL ET AL. (ed.) (Paris, 2008); AMMONIUS, *In anal. pr.*, 8, 12-13 (not p. 79, 9-12, as indicated by SCHRENK, *op. cit.*, 96).

58. Galen discussed replacing names by definitions in his lost *On Demonstration*; cf. *De methodo medendi*, vol. X, 39 Kühn, cited by R. CHIARADONNA, *Le traité de Galien Sur la démonstration et sa postérité tardo-antiqui*, in R. CHIARADONNA, F. TRABATTONI (eds.), *Physics and Philosophy of Nature in Greek Neoplatonism. Proceedings of the European Science Foundation Exploratory Workshop* (Brill, Leiden, 2009), 61. It is also an important stage in the methodology outlined in the eighth book of the *Stromata* of Clementa of Alexandria, who may have derived in from Galen; cf. HAVRDA, *op. cit.*, 360.

airesis), which descends from what is ontologically prior, the most generic genera, to what is posterior, viz. the *ultima species*: it serves to construct definitions. He also speaks of a method of synthesis which, starting out from the result of analysis, reverses the itinerary and proceeds from the ultimate species to the most generic genera. He finds both methods sketched at *Phaedrus* 265c-e.

We begin to glimpse the relevance of these Platonic methods to the Aristotelian doctrine of demonstration, in which—at least according to the interpretation of the ancient commentators—the achievement of a definition is a prerequisite for demonstration properly so called.⁵⁹ There is an important difference between the accounts of Galen and Chalcidius, however: for Galen the analytic/dihairetic method constitutes a path from above to below, while synthesis is a path from below to above; as we saw, the reverse was true for Chalcidius. If pressed, of course, the Latin author could invoke his doctrine that what seems to us to be prior or higher is in fact posterior or lower, and *vice versa*.

Elsewhere, Galen himself claims that he himself has made use of these methods. In his treatise *On the differences between pulses* (vol. 8, p. 601-606 Kuhn), while criticizing Archigenes⁶⁰ and his followers, he boasts that he has created a diagram illustrating the 27 three-dimensional and nine one-dimensional differences of pulses:

59. Cf. *APo* 75b31; 90b23: αἱ ἀρχαὶ τῶν ἀποδείξεων ὁρισμοί. Aristotle's actual attitude to the relationship between definition and demonstration is, of course, highly complex and controversial; cf. for instance J. MANSFELD, *op. cit.* 1992, 330; O. HARARI, *Knowledge and demonstration: Aristotle's Posterior analytics* (Kluwer, Dordrecht/Boston, 2004); M. DESLAURIERS, *Aristotle on definition*, (Brill, Leiden/Boston, 2007). But the ancient commentators take for granted that definitions are the principles of demonstration; in particular, as Philoponus points out (*In Apo*, p. 334, 16-17 Wallies), the middle term in a demonstrative syllogism is a definition. On the link between definition and demonstration, cf. THEMISTIUS, *In APo*, 21, 25-28; *In de an.*, 5, 9; PROCLUS, *In Parm.*, 980, 34-982, 13; PHILOPONUS, *In APo* 109, 9-110, 2; 344, 6-346, 29; PHILOPONUS, *In de an.*, 231, 13-232, 20; OLYMPIODORUS, *In Gorg.*, 5, 2; ELIAS, *In Isag.*, 9, 24; DAVID, *In Isag.*, 90, 3-4; EUSTRATIUS, *In APo II*, 51, 16-22; 55, 15-56, 4; 104, 18; EUSTRATIUS, *In EN I*, 77, 17, etc. Cf. THOMAS AQUINAS, *In APo II*, Lectio 2, 9: Definitiones enim sunt principia demonstrationum, ut in primo habitum est.

60. Archigenes of Apamea was a well-known Eclectic physician in the reign of Trajan.

How is it, though, that none of them shows he has made use of the dialectical method for the enumeration of the multitude in each genus, or the synthetic method for the discovery of first things? Yet only dialecticians can use these methods, sometimes rising up from individuals, limitless in multiplicity, to that one, first genus of all, through the intermediary generic and specific differences, sometimes again going from that <genus> to what is unlimited through the same intermediary things. But no one has dared to verify by any method the number either of the primary genera or of the differences that come about from its division. Yet we have done this...

Here, Galen once again displays his familiarity with a two-fold path, analytic and synthetic, although here again he uses the term ‘dihairetic’ or ‘divisive’ instead of ‘analytic’. Dihairetics is used to break genera down into their species, while synthesis serves to find the highest genera. Synthesis starts out from individuals—Chalcidius would say, from posterior things—and proceeds through the intermediary differentiae, both specific and generic, until it reaches the highest genus; whereas dihaeretis/analysis starts out from the highest genus and proceeds, by way of the same differentiae (also, presumably, this time examined in the reverse order), toward individuals.

Galen also invokes the methods of analysis and synthesis at length in his *On the Passions and Errors of the Soul*, p. 78 f. De Boer, but precisely what they consist in is far from clear.⁶¹ Analysis, at any rate, seems to involve subsuming a specific case under a rule that has already been demonstrated deductively; synthesis is then used, covering the same itinerary in the opposite direction. It may well be that Galen discussed the methods of analysis and synthesis in his influential *On Demonstration*, but since this work is lost, we are reduced to speculation on this point.⁶²

61. On this difficult passage, see N. CHIARADONNA, *op. cit.* 2008, 23 n. 40; HAVRDA, *op.cit.*, 366, with further literature.

62. On the *De Demonstratione* see especially HAVRDA, *op. cit.*; N. CHIARADONNA, *op. cit.* 2009. A new edition of the surviving fragments is in preparation by M. Rashed and R. Chiaradonna.

4.2 Celsus and Origen

In his *Contra Celsum*, written around 250, Origen quotes from the *Alêthês logos* of the pagan philosopher Celsus, written around 177 and hence contemporary with Galen. In the seventh book of his work, Origen quotes Celsus on Plato as an authority on theology, citing the famous remark from the *Timaeus* (28c) that the father and maker of the universe is hard to find and impossible to reveal to everyone even when found. Celsus continues:⁶³

You can see how the path of truth is sought by prophets and philosophers, and how Plato knew that it is ‘impossible’ for all to take this path. But since for this reason *it has been discovered by wise men how we might discover some concept of the unnamable and first, which would make him manifest either through synthesis <of things> above other things⁶⁴ or by analysis from them or by analogy*, I wish to teach about what is otherwise ineffable, but I would be amazed if you, who are fully bound to the flesh and see nothing pure, were able to follow.

Celsus thus knows a triad of means for knowing the unknowable God: synthesis, analysis, and analogy. Unfortunately, he gives us few details of these methods, or at least Origen has not transmitted any further explanation of them.⁶⁵ A bit further on, however, Origen, summing up Celsus’ beliefs, draws a parallel between

63. ORIGEN, *Contra Celsum*, VII, 42; H. DÖRRIE, M. BALTES (eds.), *Der Platonismus in der Antike. Grundlagen – System – Entwicklung. 7, Die philosophische Lehre des Platonismus. Theologia Platonica. 1, Bausteine 182–205: Text, Übersetzung, Kommentar*, (Frommann-Holzboog, Stuttgart-Bad Canstatt, 2008) 76–78.

64. *Hê sunthesis hê epì ta alla*: “auf die Wege der Addition zu allem anderen hinzu” H. DÖRRIE, M. BALTES, *op. cit.*, 79. For FESTUGIÈRE, *op. cit.*, 120ff. this method of synthesis is equivalent to the *via eminentiae* found elsewhere in the Middle Platonic tradition, while for Krämer (1964, 105) it instead a reversal of the method of abstraction, viz. a *via positionis* or *additionis*.

65. In fact, Origen quotes Celsus on the method of analogy at VII, 45, but appears to have left out precisely Celsus’ description of the methods of synthesis and analysis; cf. Festugière 1954, 117 n. 3.

the latter's three methods of knowing God and those used by geometers:⁶⁶

Celsus thinks that God can be known either by synthesis above other things (*hê sunthesis hê epi ta alla*), which analogous to what is called synthesis among the geometers, or by analysis from other things, or by that analogy which is analogous to the analogy among them [sc. the geometers].

Among modern scholars, Origen's interpretation was unceremoniously dismissed by Festugière,⁶⁷ who claimed that the geometrical methods of analysis, synthesis and analogy were inapplicable to the search for God. In view of the evidence from Chalcidius and Galen we have examined here, however, and the texts we shall study below, Festugière's position seems highly debatable.

4.3 Clement of Alexandria (c. 150-c. 215)⁶⁸

Like Celsus, Origen's teacher Clement of Alexandria was also aware of analysis as a process of abstraction:⁶⁹

...progressing to the first intellection through analysis, taking our starting-point by analysis from the things that underlie

66. Origen, *Contra Celsum*, 7.44: Κέλσος μὲν οὖν ἦτοι τῇ συνθέσει τῇ ἐπὶ τὰ ἄλλα ἀνάλογον τῇ παρὰ τοῖς γεωμέτραις καλουμένη συνθέσει ἢ τῇ ἀπὸ τῶν ἄλλων ἀναλύσει ἢ καὶ ἀναλογία ἀνάλογον τῇ παρὰ τοῖς αὐτοῖς ἀναλογία οἴεται γινώσκεσθαι τὸν θεόν.

67. Festugière 1954, 120: "Origène voit ici les trois méthodes mathématiques d'analyse, de synthèse, de proportion. Or cette exégèse est fausse".

68. For the dates of the *Stromata* (*terminus post quem* 192?), see for instance Havrda 2011, p. 373 n. 121.

69. Clement of Alexandria, *Stromata*, 5.11.71.2-4: τὸν δὲ ἐποπτικὸν ἀναλύσει ἐπὶ τὴν πρώτην νόησιν προχωροῦντες, δι' ἀναλύσεως ἐκ τῶν ὑποκειμένων αὐτῷ τὴν ἀρχὴν ποιούμενοι, ἀφελόντες μὲν τοῦ σώματος τὰς φυσικὰς ποιότητας, περιελόντες δὲ τὴν εἰς τὸ βάθος διάστασιν, εἴτα τὴν εἰς τὸ πλάτος, καὶ ἐπὶ τούτοις τὴν εἰς τὸ μῆκος· τὸ γὰρ ὑπολειφθὲν σημεῖόν ἐστι μονὰς ὡς εἰπεῖν θεσιν ἔχουσα, ἥς ἐὰν περιέλωμεν τὴν θέσιν, νοεῖται μονάς, εἰ τοίνυν, ἀφελόντες πάντα ὅσα πρόσσεσι τοῖς σώμασιν καὶ τοῖς λεγομένοις ἀσώματοις, ἐπιρρίψαιμεν ἑαυτοὺς εἰς τὸ μέγεθος τοῦ Χριστοῦ κακεῖθεν εἰς τὸ ἀχανὲς ἀγιότητι προῖοιμεν, τῇ νοήσει τοῦ παντοκράτορος ἀμῇ γέ πη προσάγοιμεν <ἄν>, οὐχ ὅ ἐστιν, ὃ δὲ μὴ ἐστι γινώσαντες.

him, subtracting physical qualities from the body, stripping off extension in the direction of depth, then that which is in the direction of breadth, and after these that which is in the direction of length. The point that is left over is a monad, which, as it were, has position,⁷⁰ and if we remove its position, one conceives of the monad. If, then, having subtracted everything that pertains to bodies and the so-called incorporeals, we were to hurl ourselves into the magnitude of Christ (...) we would somehow draw near to the intellection of the Pantocrator, knowing not what He is, but what He is not.

The Middle Platonist philosopher Alcinoos is familiar with a similar scheme.⁷¹

Let the first thinking [sc. of God] be that which takes place by abstraction [sc. of the divine epithets previously enumerated], as we have thought of the point by abstraction (*kata aphairesin*) from the sensible, thinking of a surface, then a line, and finally the point.

Here, as in several other Middle Platonic texts,⁷² the process of analysis is conceived along the lines of a reversal of the emanationist scheme current in Plato's Academy: from One or the Monad emanates the Indivisible Dyad;⁷³ from the combination of the One and the Indivisible Dyad there results number; then, in a process

70. *monas* (...) *thesin ekhonta*. Cf. Arist., *Phys.*, B. 227 a 27; *Metaph.* 5.6, 1016b10ff.; Philoponus, *In de an.*, 166, 31; Thomas Aquinas, *1 anal.* 41 a; *1 anim.* 11 b; *1 met.* 2 i; cf. *5 met.* 8 c; *5 phys.* 5 i.

71. *Didaskalikos*, X, 5, p. 165, 16ff. Hermann; H. DÖRRIE, M. BALTES, *op. cit.*, 190.3. This text, with its presentation of the three methods of approach to God which are almost, but perhaps not quite, identical to those of Celsus, has often been studied: for bibliography see Whittaker's Budé edition p. 106-107 n. 203; H. DÖRRIE, M. BALTES, *op. cit.*, 377ff.

72. Especially Plutarch, *Quaestiones Platonicae* 3, 1001F-1002A. Cf. also Philo, *De somn.* I, 186f.

73. One of several ways this is envisioned as happening is by the process of *episunthesis*, "addition" or "combination"; cf. H-J. KRÄMER, *Der Ursprung der Geistmetaphysik. Untersuchungen zur Geschichte des Platonismus zwischen Platon und Plotin*, (Schippers, Amsterdam, 1964) 320.

whereby dimensions are successively added, the point, line, surface, and solid body emerge, followed by qualities. To rise back up from the sensible world in which we find ourselves, we mentally reverse this process, removing all qualities to reach geometrical body, then removing depth, breadth and length to arrive at the point. When we remove position from the latter, we have arrived at the intelligible Monad, which Clement somewhat clumsily identifies with Christ.

Texts such as those from Clement, Alcinoos, and others, suggest that we have to do here with an Old Academic scheme of thought, which may go back to the unwritten doctrines of Plato himself.⁷⁴ To quote Krämer,⁷⁵

The threefold division of theological methods in Celsus VII 42 (...) and, with slight divergence, in Albinos Didask. X 5, p. 165, 14ff. H. allows us to recognize that an ‘additive’-positive (*sunthesis*, cf. Alb. Did. c. V 4, p. 157, 9 ff. H.) and an ‘abstractive’-negative procedure (*analysis-aphairesis*) completed one another in an antithetical way.

If Krämer is right, then just as in the mathematical method, where an initial “inductive” analytical ascent from a supposition to an agreed-upon principle is then confirmed by a “deductive”/demonstrative, synthetic descent from that principle to the question under investigation, we have in the Middle Platonic theological tradition a method by which one uses analysis to start out from the sensible world and, by a method of abstraction similar to that used to conceive of a mathematical point, arrives at the intelligible or the One beyond it. In turn, this analytical ascent can be confirmed by “synthetic” descent from the highest principles, in which one knows God by contemplating the world that derives from Him.⁷⁶

74. H-J. KRÄMER, *op. cit.*, 106f.; 118; 343ff. On analysis in Plato, see for instance S. MENN, *Plato and the Method of Analysis*, “Phronesis” 47 (2002) 193-223.

75. H-J. KRÄMER, *op. cit.*, 350. My translation.

76. As examples of this *kata sunthesin* method, Krämer cites Plotinus V 4, 1, 1; V 5, 6, 12-34; V 5 13, 9-18; DAMASCIUS, *De princ.* c. 28 p. 51, 1 Ruelle; Ps.-Dionysius, *De div. nom.* II; *myst. theol.* II f.

Returning to Clement of Alexandria, we find that he was also familiar with the method of synthesis as complementary to that of analysis, which he discusses, for instance, in the context of his description of dialectic in the eighth book of his *Stromata*:⁷⁷

*Geometrical analysis and synthesis resemble dialectical division and definition, and we revert (anatrekhein) from division toward what is simpler and more principal. Indeed, we divided the genus of the thing being sought into the species inherent within it: for instance, in the case of man, we divide 'animal', which is the genus, into the species that appear, viz. mortal and immortal, and thus, always cutting the genera that seem to be compound into the simpler species, we come to what is sought, which no longer allows a cut. For once we have divided 'animal' into 'mortal' and 'immortal', then, however, 'mortal' into 'terrestrial' and 'aquatic', and again 'terrestrial' into 'winged' and 'footed', and dividing in this way the species proximate to what is being sought, which also contains what is being sought, we arrive, by cutting, at the simplest species, which contains nothing other than what is being sought. For once again, we divide 'footed' into 'rational' and 'irrational'; then, selecting from the species taken from the division those that are proximate to 'man' and putting them together into one *logos*, <we have> the definition of man, which is 'a mortal, terrestrial, footed, rational animal'.*

Here, Clement agrees with Galen in affirming that analysis is analogous to Platonic *diairesis*, but he spells out what Galen had only hinted at: how one uses analysis and synthesis to construct a definition. The algorithm for finding the definition of man,⁷⁸ for instance, is relatively straightforward: starting with the genus, animal, we divide it dichotomously into mortal and immortal. From these two options, we then choose the one most appropriate to man, in this case mortal, which we again divide dichotomously. We continue

77. CLEMENT OF ALEXANDRIA, *Stromata*, 8.6.18.1-19.1.

78. Cf. ALCINOOS, *Didask*, V, p. 157, 27-36.

the procedure until we arrive at the simplest species, in this case ‘rational’. To construct our definition, we simply add to the genus the differentiae we have chosen at each stage, ending up, in the case of ‘man’, with ‘mortal, terrestrial, footed, rational animal’.

Clement thus has the merit of making explicit the similarity between dialectical division and definition, on the one hand, and geometrical analysis and synthesis on the other. As for Galen, from whom Clement may have derived his material, analysis is similar to Platonic *diairesis*, while synthesis is analogous to putting together the results of this dihairetic procedure in order to construct a definition. He thus reminds us of the mathematical notion of analysis and synthesis we have studied above.⁷⁹

5. ALCINOOS

As Van Winden⁸⁰ and Waszink⁸¹ pointed out long ago, the closest Greek parallel to our Chalcidius passage comes from chapter 5 of the *Didaskalikos* of Alcinoos, a Middle Platonist philosopher roughly contemporary with Galen but about whom, as about Chalcidius, we know almost nothing.

In this chapter of his philosophical handbook, Alcinoos is describing the task of dialectics, which investigates the essence of each thing in two ways: either from above (*anôthen*) by division and definition, or from below (*katôthen*) by means of analysis (*analutikôs*).⁸² Accidents, for their part, are also to be investigated in one of two ways: either from the things that are contained, by induction (*di*’

79. It may well be the case, as argued persuasively by HAVRDA, *op. cit.*, that the material in the controversial eighth book of Clement’s *Stromata* in fact derives from Galen’s lost work *On Demonstration*. Yet the scheme is hardly original with Galen, as I hope to show in this paper.

80. J. C. M. VAN WINDEN, 1959. *Chalcidius on Matter. His Doctrine and Sources. A Chapter in the History of Platonism*, “Philosophia Antiqua” 9 (1959).

81. J. H. WASZINK (ed.), *Timaeus a Calcidio translatus commentarioque instructus*, (Instituti Warburgiani/E. J. Brill, Londinii/Leidae, 1962) (Plato Latinus, 4; Corpus Platonium medii aevi; Corpus philosophorum medii aevi).

82. Cf. Chalcidius on the *resolutio quae <ex> posterioribus a praecedentium indaginem gradatim peruenit*.

epagôgês), or from the things that contain, by syllogism (*dia sullogismou*). There are therefore five parts of dialectic: division, definition, analytics, induction, and syllogism.⁸³

Discussing these five subdivisions in turn, Alcinoos begins with division (*diairesis*), which he breaks down into five sub-headings,⁸⁴ and then goes on to quickly deal with definition, which, as we have seen from Galen and Clement, is obtained as a result of division.

Alcinoos now moves on to analysis.⁸⁵ There are, he tells us (p. 157, 11ff. Hermann), three kinds:

1. That which is an ascent (*anodos*) from sensible things to the first intelligibles.

2. That which is an ascent (*anodos*) from things that are shown or suggested to the indemonstrable, immediate premises; and finally

3. That which ascends from a hypothesis to the non-hypothetical principles.

Of these three types, the first represents the familiar ascent from the intelligible to the sensible as described in Diotima's speech in Plato's *Symposium* (201a6-e1). In the third type of analysis, one begins by supposing what is under investigation, and then examines what follows from this supposition. If one subsequently needs to account for this hypothesis, one proposes another hypothesis and sees whether the first is consistent with the second, and one repeats the procedure until a non-hypothetical principle (*arkhê*) is reached. This is clearly a stripped-down account of the methodology set forth at the end of *Republic* VI.

83. Subsequent Neoplatonists will divide dialectic into division, analysis, definition and demonstration; cf. below.

84. These are (1) division of genus into species; (2) of whole into parts; (3) of a word into its meanings; (4) of accidents into their substrates; (5) of substrates into accidents. Boethius (*De divisione*, ad init.) reports an almost identical division, which he adopts from Andronicus by way of Porphyry; cf. Porphyry fr. 169F Smith. One first takes the genus of the thing to be defined, and then cuts it up (Greek *temnein*) according to its proximate differences: for instance, wishing to define 'man', one takes its genus 'animal', then divides 'animal' into 'rational' and 'irrational' or 'mortal' and 'immortal'. Finally, by adding these proximate differences to 'animal', one obtains the definition of 'man'.

85. Cf. M. MARÓTH, *Ibn Sina und die peripatetische «Aussagenlogik»*, (Brill, Leiden, 1989) 102f.

It is Alcinoos' second type of analysis that most interests us here. Here, one assumes what is being sought (*to zêtoumenon*) and considers the things that are prior to it, demonstrating them by ascending from posterior to prior things⁸⁶ until one arrives at what is primary and agreed upon (*homologoumenon*).⁸⁷ From this point, Alcinoos adds, one will descend to the thing being sought by means of the synthetic method.⁸⁸

The parallels we can observe between Alcinoos' account of his second type of analysis and the accounts we have studied of analysis and synthesis in Greek geometry make it clear that Alcinoos is not following Aristotelian methodology here, any more than Chalcidius is, but the synthetic/analytic method as codified by the Greek geometers.⁸⁹ It becomes all the more likely, therefore, that Origen was right, and that Celsus too was referring to geometrical practices in his discussion the ways to achieve knowledge of God.

For Alcinoos, assuming the proposition under investigation, for instance that the soul is immortal, analysis proceeds through a series of "consequences" until some proposition is reached that is agreed upon by all: what the Arabo-Islamic geometers would later call "the knowns". Synthesis then starts out from this agreed-upon proposition and covers the same steps in reverse. Ideally, one ends up in this way with a demonstration whose conclusion is that the soul is imperishable, ungenerated, and immortal.

86. ἀπὸ τῶν ὑστέρων ἐπὶ τὰ πρότερα ἀνιόντα Alc., *Didasc.* 157, 24-25 Hermann; cf. Chalcidius: quae <ex> posterioribus ad praecedentium indaginem gradatim peruenit, quod genus probationis resolutio dicitur.

87. ἕως ἂν ἔλθωμεν ἐπὶ τὸ πρῶτον καὶ ὁμολογούμενον Alc.; ἐπὶ τι ἀληθὲς ὁμολογούμενον; schol. Eucl.; "Then we resolve it to something whose proof is already had", Hero ap. al-Nayrīzī (trans. Lo Bello); ἕως ἂν οὕτως ἀναποδίζοντες καταντήσωμεν εἰς τι τῶν ἤδη γνωριζομένων ἢ τάξιν ἀρχῆς ἐχόντων Pappus.

88. ἀπὸ τούτου δὲ ἀρξάμενοι ἐπὶ τὸ ζητούμενον κατελευσόμεθα συνθετικῶ τρόπῳ Alc.; "synthesis, that is when one begins with the known things; then one combines them until what is sought is found" Hero ap. al-Nayrīzī (trans. Lo Bello modified); τὸ ἐν τῇ ἀναλύσει καταληφθὲν ὕστατον ὑποστησάμενοι γεγονὸς ἤδη, (...) εἰς τέλος ἀφικνούμεθα τῆς τοῦ ζητουμένου κατασκευῆς Pappus.

89. A point that seems to have been missed by Whittaker in his otherwise excellent edition of ALCINOOS (Les Belles Lettres, Paris, 1990).

6. ANALYSIS AND SYNTHESIS IN THE LATER COMMENTATORS ON ARISTOTLE: ANALYSIS AS A SUBDIVISION OF DIALECTICS

6.1 *Alexander of Aphrodisias*

Like his near-contemporary Galen, Alexander of Aphrodisias (fl. 200CE)⁹⁰ knows that analysis, which is the reduction of compound things to their elements, is the converse of synthesis, which is the path from principles to their derivatives. In addition to the reduction of compound to simple bodies, Alexander tells us, there are various senses of the term ‘analysis’, including analysis of simple things into their matter and form; of a phrase into parts of speech, syllables, and finally letters; of compound syllogisms into simple ones, and of simple ones into their premises; of imperfect syllogisms to perfect ones, and of syllogisms into their figures. Alexander (or someone in his circle) also knows of a “proof (*deixis*) by analysis”⁹¹ that serves, as in Chalcidius, Celsus, and Alcinoos, for achieving knowledge of otherwise unknowable first principles:

For it is not possible for there to be a demonstration (*apodeixis*) of the first principle, but one must begin with what is posterior and manifest, and establish its nature by making use of analysis in accordance with the agreement with these things.

6.2 *Porphiry and Proclus*

In his *Commentary on Euclid*, the Athenian Neoplatonist Proclus (c. 412-485)⁹² provides a classification of analysis and synthesis within the series of mathematical proofs.

90. *In Anal pr.* 1, 7, 11ff. Cf. SCHRENK, *op. cit.*, 101-2.

91. ALEXANDER (?), *Quaestiones* 4.3-7: ἡ δειξις κατὰ ἀνάλυσιν. οὐ γὰρ οἷόν τε τῆς πρώτης ἀρχῆς ἀποδείξιν εἶναι, ἀλλὰ δεῖ ἀπὸ τῶν ὑστέρων τε καὶ φανερῶν ἀρξαμένους κατὰ τὴν πρὸς ταῦτα συμφωνίαν ἀναλύσει χρωμένους συστήσαι τὴν ἐκείνου φύσιν. The authorship of the *Quaestiones* is disputed, but one may assume that the doctrines they transmit are not too far removed from the views of the Aphrodisian.

92. *In Eucl.*, 255, 13-23 Friedlein.

In general, one should know that all mathematical proofs are either from the principles or toward the principles, as Porphyry says somewhere.⁹³ Those that are from the principles are also twofold, for they either start out from the common notions and the mere clarity of what is credible in itself, or from things that have been shown previously.

Those that go toward the principles either posit the principles or negate them. If they posit them, they are called analyses, and opposite to these are the syntheses—for it is possible to proceed in good order from those principles to what is being sought, and this is synthesis—but if they are negative they are called *reductio ad impossibile*.

Here we have a clear statement of the idea that analysis, by which, as Proclus tells us elsewhere,⁹⁴ one reduces the problem under investigation to (*epi*) a principle (*arkhê*) that everyone agrees on, is the reverse process to synthesis, by which one proceeds from the principles to the problem under investigation. Interestingly, Proclus attributes the analytic method to none other than Plato, who then transmitted the technique to a certain Leodamas.⁹⁵ This attribution is usually dismissed as mere fabulation; but our study so far has perhaps suggested there may be a grain of truth to the story. While it is unlikely that Plato *invented* analysis, it does not seem to be impossible that he adopted a method already in use among contemporary geometers, and adapted it to use in philosophy.⁹⁶

93. PORPHYRY, fr. 482, p. 552 Smith.

94. *In Eucl.*, p. 211, 19-212, 3.

95. Leodamas of Thasos; cf. Diogenes Laertius 3, 24.

96. Cf. SCHRENK, *op. cit.*, 96 and n. 11. Writing without any apparent knowledge of the doctrines of Alcinoos or later Platonists, P. TANNÉRY *La géométrie grecque. Comment son histoire nous est parvenue et ce que nous en savons. Essai critique. Première partie, Histoire générale de la géométrie élémentaire*, (Gauthier-Villiers, Paris, 1887) 112-113 (Reprint Hildesheim etc.: Georg Olms, 1998). Tannéry already suggested that *Republic* VI contained an attempt at a systematization of the method of analysis and synthesis, which consists in “remonter de l’hypothèse au principe non supposé; suivre la marche inverse du principe à l’hypothèse”. Cf. Heath, citing *Rep.* VI 510B; 511A-C. T. HEATH, *A History of Greek Mathematics. 1, From Thales to Euclid*, (Clarendon Press, Oxford, 1921); FESTUGIÈRE, *op. cit.*, 120.

6.3 *Ammonius*

Proclus' student Ammonius (c. 440-520)⁹⁷ is also familiar with philosophical analysis and synthesis. Synthesis, he writes, is the path from simple forms (such as the Beautiful in itself) toward compound things (Beauty as exemplified in the intellect, soul, and body). Analysis is the reverse path, from the form or idea within sensibles to the forms in the intelligibles. There is also geometrical analysis, which Geminus described as the discovery of a demonstration.⁹⁸

Elsewhere,⁹⁹ Ammonius speaks of a division, already well-known to his teacher Proclus,¹⁰⁰ of Platonic dialectics into division, definition, demonstration and analysis.¹⁰¹ Of these, he tells us, division divides the genus into its differences in an orderly manner, definition gives the essence of each thing from its genus and constitutive differences, while demonstration demonstrates things on the basis of their substance and of definitions. Finally, analytics analyses compound things into the simple things of which they are composed, as one analyses a phrase into nouns and verbs, then syllables, then letters. Likewise, philosophers analyze man into head, hands and feet; these into bones, flesh and nerves, and these into elements. One also analyses syllogisms into premises, and these into terms.¹⁰²

97. *In anal. pr.* I, 5, 19-31.

98. Cf. SCHRENK, *op. cit.*, 97.

99. *In Porph. Isag.*, 34, 15ff.

100. Cf. G. VAN RIEL, *Damascius, Commentaire sur le Philèbe de Platon*,; en collab. avec C. MACÉ, J. FOLLON (Les Belles Lettres, Paris, 2008) 18 n. 2, citing Proclus, *In Crat.* 2-3, pp. 1.10-2.1; *In Parm.*, I.649.14-651.7; 5.980.3-982.40; 1003.6-29; *In Tim.* I, p. 276.1014; *Theol. plat.* I 9, p. 40.5-10; *De prov.* 30.1-4. Additional texts are cited by Westerink in the Introduction to his Budé edition of the anonymous *Prolegomena to Philosophy* (Paris: Les Belles Lettres, 1990), p. lxxiv.

101. Cf. Alcinoos' division, discussed above, into division, definition, analytics, induction, and syllogism. One example from Arabo-Islamic philosophy: in opening chapter of the *Liber Introductorius*, (A. NAGY, *Die Philosophischen Abhandlungen des Ja'qūb ben Ishāq al-Kindī*, (Aschendorffschen Buchhandlung, Münster, 1897) 41, long attributed to Al-Kindī but now recognized as a Latin translation of the treatise *Fī Anūlūṭīqā al-Ulā* of the 10th-century *Encyclopaedia of the Brethren of Purity* (Iḥwān al-Ṣafā'), we read of the *viae per quas ambulauerunt philosophi, in illis disciplinis, in quibus sua inquisitio fuit de cognitione certitudinis rerum, comprehenduntur in quatuor speciebus, scilicet divisione et resolutione, definitione et demonstratione*. Note the different order of enumeration, with analysis/resolutio being promoted from last to second place. I thank Dr. Irene Caiazza for pointing out this text to me.

102. Cf. SCHRENK, *op. cit.*, 101.

6.4 *Philoponus*

As Richard Sorabji has pointed out (2004, Vol. 3, p. 268-9), some of the most perspicuous accounts of the method of analysis and synthesis comes from Ammonius' student John Philoponus (c. 490-570). Discussing geometrical analysis,¹⁰³ he tells us that it is the discovery of the premises leading to some true conclusion. Let's assume, for instance, that the triangle before us is equilateral: we are to discover the premises leading up to this conclusion. As in the geometrical occurrences we studied above, so for Philoponus synthesis, the inverse process to analysis, begins with something agreed-upon (*to homologoumenon*) and stops at what is sought (*to zêtoumenon*). Analysis, by contrast, takes what was previously sought as agreed-upon, whereupon we seek out the premises by which this conclusion has been established, until we arrive at things that are agreed upon and represent the principles of geometry.

Elsewhere (*In APo*, p. 334, 19ff.) Philoponus discusses the reasons why Aristotle's *Posterior Analytics* is entitled "analytics". There is one species of analysis, studied in the *Prior Analytics*, in which we analyze syllogisms into their moods and figures. There is another species, however, in which we analyze the known object into the principles and causes from which it has its being and its being known, and this is the sense in which demonstration is called analysis:¹⁰⁴

For it is by analysis that its principles are discovered, as we ascend from effects, which are prior *to us*, to the causes, which are prior *by nature*.

With its depiction of analysis as a process starting out from what is more manifest *quoad nos* and ascending to the discovery of causes and principles, while are more knowable *per se*, Philoponus' explanation harks back to Calcidius, who wrote about a century earlier.

103. PHILOPONUS, *In APo*, 162-173.

104. PHILOPONUS, *In APo*, 335: ἐξ ἀναλύσεως γὰρ ἡμῖν αἱ ἀρχαὶ ταύτης εὐρίσκονται ἀπὸ τῶν ἡμῖν προτέρων αἰτιατῶν ἀνιοῦσιν ἐπὶ τὰ τῇ φύσει πρότερα, ἡγουν τὰ αἷτια.

Yet it also points forward the later Latin tradition, thereby offering a good illustration of the surprising persistence and longevity of this philosophical theme. Albertus Magnus, for instance, writes as follows in the preface to his own commentary on the *Posterior Analytics*:¹⁰⁵

There cannot help but be a twofold analysis, viz. of the concluded thing into the principles and causes by which it is concluded, and of the syllogism which is already collected and constituted into its formal principles...

Philoponus proceeds to illustrate his remarks by the well-known example of the eclipse. We first know through our senses that the moon undergoes eclipse, after which discursive thought (*dianoia*) finds the cause of this phenomenon. Analysis then constructs the following syllogism, which is an example of the ascent from caused to cause: “The moon is eclipsed, what is eclipsed intercepts the sun’s light, therefore the moon intercepts the sun’s light”. Demonstration, obtained by means of synthesis, then takes place by descending from causes to caused: The moon intercepts the sun’s light, what intercepts the sun’s light undergoes eclipse, therefore the moon undergoes eclipse.¹⁰⁶

Thus, Philoponus elaborates upon the preceding meanings of analysis we have already studied, coming up with a pair of methodological approaches that resemble what we know today as induction and deduction. We have seen above that analysis was assimilated to Platonic *diairesis*, the art of dividing Forms by which we arrive at definitions. Now, Philoponus, perhaps following Pappus, makes it explicit

105. ALBERTUS MAGNUS, *In APo*, vol. I, 460a Borgnet: Nec potest esse nisi duplex resolutio, scilicet rei conclusae in principia et causas per quas concluditur, et syllogismi collecti jam et constituti in principia formalia.

106. Similarly, we may see the earth shaken, and say, beginning with the analysis, “The earth is shaken, but when it is shaken pneuma is enclosed in the earth’s hollows and caves. Pneuma is enclosed in the earth”. We then proceed with the demonstration: “Pneuma is enclosed within the earth, when pneuma is enclosed an earthquake takes place, therefore an earthquake takes place in the earth”. On the fact that synthesis, as the reverse procedure to analysis, is equivalent to demonstration in mathematics and logic, see PAPPUS, *Collectiones*, I, 144, 22 Hultsch, cited by M. MARÓTH, *op. cit.*, 102.

that synthesis, the inverse method to analysis, corresponds to demonstration. Analysis infers from effect to cause, from the *consequens* to the *antecedens*, while synthesis infers from cause to effect, from *antecedens* to *consequens*. We thus have the following correspondences:

analysis	context of discovery	induction (from effect to cause)
synthesis	context of proof	deduction (from cause to effect)

Thus, by the end of Antiquity, we have in the analysis/synthesis schema a model of scientific thought which presents strong analogies to distinctions that have played a crucial role in modern philosophy of science: that between context of discovery (analysis) and context of proof (deduction), on the one hand, and on the other that between induction and deduction. These distinctions would have lasting impact on subsequent Arabo-Islamic and Latin Scholastic thought.¹⁰⁷

6.5. *Elias*

Finally, one of the most interesting ancient Greek texts concerning synthesis and analysis comes from Elias,¹⁰⁸ one of the last known Neoplatonic commentators on Aristotle in Late Antiquity, in the Prologue he placed before his commentary on the *Isagogê* of Porphyry:

107. On analysis as an *ars inveniendi* see, for instance, THĀBIT IBN QURRA, *On how to be able to determine the construction of geometrical problems*, in RASHED (ed.), *op. cit.*, 742-764, which the editor describes (loc. cit., 690) as “the first work on the method of invention”. See also HASNAWI, *op. cit.*, 29-30.

108. First called to attention, as far as I know, by Schrenk. Unfortunately, this author’s mistranslation of the fourth dialectical problem *dia ti estin* as “through what it is” rather than “for what <reason> it is” (104) prevented him from seeing this problem’s relation to the Aristotelian final cause. Whether this Prologue to Porphyry’s *Isagoge* (*Eliae in Porphyrii Isagogen et Aristotelis Categorias*, A. BUSSE (ed.), “CAG” 18/1 (1900) 1-104) is really by Elias or should rather be attributed to David is a matter of scholarly debate, but need not be entered into here (we know almost nothing about either figure, except that they probably date from the late 6th century CE). For a summary of the controversy, see R. GOULET, *Élias*, “Dictionnaire des Philosophes Antiques” III, (Presses du CNRS, Paris, 2002) 57-66.

But [the *Isagoge* also contributes] to the dialectical methods. There are four of these: divisive, definitional, demonstrative, and analytical, since the dialectical problems are also four in number: if it is, what it is, what it is like, and why it is.¹⁰⁹ “If it is” is analogous to the divisive method (for “if it is” falls, at any rate, under division), while “what it is” <falls under> the definitional method <for definitions present the “what-it-is”, while the “what it is like” falls under the demonstrative method>¹¹⁰ (for all demonstrations are of what a thing is like. The “if it is” is seldom controversial, and there is no demonstration of the definition), while the “why it is” is analogous to analytics, for each of them is the discovery of a cause. (...) the demonstrative method starts out from causes and first things, while this one <sc. analysis> starts out from effects and secondary things: for analysis is nothing other than a reverse demonstration, whence <it is called> ‘analysis’ insofar as it is a reverse solution of the matter at hand.¹¹¹

This text deserves an ample commentary, for which this is not the place. Suffice it for the moment to say that it echoes several themes we have already glimpsed, and at the same time paves the way to the future. The term “synthesis” is not explicitly mentioned in it, but we can see from the last phrase of our quotation that it is here being tacitly identified with demonstration. Elias discusses the familiar quaternity of dialectical methods in the order that, as we have seen, had been canonical at least since Proclus: division, definition, demonstration, and analysis, and allows us to understand why they are enumerated in that order: division allows the construction of defini-

109. Elias has introduced these four dialectical questions earlier in his *Commentary on the Isagoge*: cf. p. 3, 5ff. Busse, where he speaks of the “dialectical law (*nomos dialektikos*)” which states that in the case of all arts and sciences, one must investigate four problems: if it is, what it is, what it is like, and why it is”. They go back at least to Proclus (*In Alcibiadem*, 275, vol. 2, p. 316, 1-4 Segonds), who off-handedly attributes them to Aristotle (*Eirêtai pou kalôs hupo tou Aristotelous*).

110. Since the Greek text presents a lacuna here, I have translated Busse’s conjecture in his *apparatus criticus*.

111. Cf. Pappus and Ibn al-Ṭayyib, quoted above.

tions, which then represent the starting-point for demonstration.¹¹²

The significance of this text for the future lies in its association of the four species of dialectic with the four “dialectical questions”, which are similar to,¹¹³ yet significantly different from, the four questions enumerated by Aristotle at the beginning of Book II of the *Posterior Analytics*. These four dialectical questions, whose origin is in fact, I believe, to be sought in part in the rhetorical tradition of Hermogenes,¹¹⁴ are subsequently associated with the four Aristotelian causes:

Arist., <i>APo</i> 89b23-25	Elias, <i>In Porph. Isag.</i>	species of dialectic	Aristotelian causes
<i>to hoti</i> (the fact)	<i>ei esti</i> (if it is)	division	efficient cause
<i>to dioti</i> (the why)	<i>ti estin</i> (what it is)	definition	formal cause
<i>ei esti</i> (if it is)	<i>hopoion ti esti</i> (what sort of thing it is)	demonstration	material cause
<i>ti estin</i> (<i>what it is</i>)	<i>dia ti estin</i> (why it is)	analysis	final cause

112. AMMONIUS, *In Porph. Isag.*, p. 34, 17-36, 19. Cf. SCHRENK, *op. cit.*, 103-104.

113. The Commentators occasionally attribute this “Elias series” to Aristotle, and more specifically to the *Posterior Analytics*: cf. OLYMPIODORUS, *In Gorg.*, 2, 8, 8 (there is no mention of the disparity in R. JACKSON, K. LYCOS AND H. TARRANT, transl., *Olympiodorus, Commentary on Plato’s Gorgias*, (Brill, Leiden, 1998) 74); ASCLEPIUS, *In Metaph.*, 448, 13-14.

114. Hermogenes of Tarsus (latter half of the 2nd cent. CE) mentions at least the first three questions, in the same order as Elias. Cf. *Peri staseôn*, in M. PATILLON (ed.), *Corpus Rhetoricum. 2, Hermogène, Les états de cause*, 2, 14, 17-18: whether a thing is (*ei esti*) is the domain of conjecture (*stokhasmos*); what it is (*ti esti*) is examined in the definition, and what kind of thing it is (*hopoion ti est*) is studied in the remaining states of cause. Later commentators on Hermogenes add the fourth question, *dia ti estin*.

It is this later scheme, in the form given it by such late Greek commentators as Elias, David and Eustratius, that is then taken up the medieval Jewish and Islamic philosophical tradition.¹¹⁵ But that is another story.

7. CONCLUSION

To sum up this long and complex journey: there appears to have existed, at least as early as the time of Plato's Academy, a mathematical method of analysis and synthesis in which one started out by supposing as solved the problem under discussion (*to zêtoumenon*), and then traced back the argumentative steps to an axiom, principle, or notion accepted by all (*to homologoumenon*). One then confirmed this result by starting out from the *homologoumenon* and reversing one's steps, in a downward, synthetic movement toward the *zêtoumenon*. Perhaps as early as Plato's time, this methodology was adapted to the needs of philosophy, where it was interpreted as a means for coming to know the highest principles. According to this method, one starts out from the sensible world and abstracts from it the various spatial dimensions that constitute it: just as one finds a mathematical point, and then an intelligible monad, by mentally abstracting dimensions, from body, plane, and line, so we can find God, the Intelligible world, or even matter (which is also a principle) by a similar process of abstraction, and we can then confirm this process by starting out from the principle thus obtained and putting back together what we had taken apart, confirming the nature of God or the Intelligible from the sensible, extended features of the world that emerge from them.

In the course of a millennium of thought, both the mathematical and the philosophical versions of the analytic/synthetic method underwent important modifications, becoming associated with a

115. See the opening chapter of the *Book of Definitions* by Isaac Israeli (c. 832-c. 932), with the parallel passages cited by A. ALTMANN, S. M. STERN, *Isaac Israeli. A Neoplatonic philosopher of the early tenth century. His works translated with comments and an outline of his Philosophy*, (U. of Chicago Press, Chicago, 2009) from al-Kindī, Ibn al-Ṭayyib, al-Ṭabarī, Miskawayh, al-Khuwārismi, al-Jāhiz.

number of new contexts: Platonic and Aristotelian dialectic, with their division into *diairesis*, definition, and demonstration; the contrast between methods of discovery and methods of proof, induction and deduction; and perhaps the interpretation of the rhetoric of Hermogenes. As time went on, it is quite possible that the philosophers themselves who transmitted the evidence we have of this complex of thought were unaware of its origins. Thus, when Chalcidius and Alcinoos speak of analysis as a process of abstraction used for discovering ultimate principles, and again as a subsection of Platonic dialectics, and again, this time paired more explicitly with the corresponding downward path of synthesis, as a method of discovery, they may be uncertain as to how these seemingly different processes are, or at least, were at one point, connected; hence the occasional confusion and vagueness of their expositions, and their attempt to clarify the confusion by introducing, for instance, several subspecies of analysis. Matters were even less clear for such thinkers as Albertus Magnus and Thomas Aquinas, who had considerably less knowledge of the Platonic/Academic origins of these notions than did Alcinous or Chalcidius. By picking up on hints in the sources to which they did have access, they were able, thanks to their own philosophical creativity, to render the notions of analysis and synthesis fruitful once again. Yet the extent of their creativity and originality can be properly measured only against the background of the history of these notions.¹¹⁶

116. I would like to thank my anonymous readers for their valuable suggestions.